Calc BC HW Quiz 2-15

Name: Block: Seat:

Use the following choices to answer the questions below:

(a) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{x^{2n}}{(2n)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ $(for\ x \in \mathbb{R})$

(b) $1 + x + x^2 + x^3 + x^4 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n$ $(for\ -1 < x < 1)$

(c) $1 - x + x^2 - x^3 + x^4 - \cdots + (-x)^{n-1} + \cdots = \sum (-1)^n x^n$ $(for\ -1 < x < 1)$

(d) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $(for\ x \in \mathbb{R})$

(e) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ $(for\ x \in \mathbb{R})$

(f) $x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^{n-1}\frac{x^n}{n} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n}$ $(for\ -1 < x < 1)$

(g) $x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{(-1)^n x^{2n+1}}{2n+1} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$ $(for\ -1 \le x \le 1)$

1. $\cos(x) =$ A

2. $\sin(x) =$ E

3. $e^x =$ D

4. $\ln(1+x) =$ F

5. $\arctan x =$ G

6. $\frac{1}{1-x} =$ B

7. $\frac{1}{1+x} =$ C

 Note $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{(2n-1)}}{2n-1}$

8. Use the series for $f(x) = \arctan x$ to approximate the value of $\arctan \frac{1}{2}$ so that $\left|f\left(\frac{1}{2}\right) - P\left(\frac{1}{2}\right)\right| \le \frac{1}{1000}$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} + \cdots + \frac{(-1)^{n+1} x^{2n-1}}{2n-1} - \cdots$$

$$\arctan \frac{1}{2} = \frac{1}{2} - \frac{1}{2^3 \cdot 3} + \frac{1}{2^5 \cdot 5} - \frac{1}{2^7 \cdot 7} + \frac{1}{2^9 \cdot 9}$$

$$= \frac{1}{2} - \frac{1}{24} + \frac{1}{160} - \frac{1}{896} + \frac{1}{4608}$$

By Alt Series Remainder Thm

Since $|a_5| = \frac{1}{4608} < \frac{1}{1000}$

The sum of the first 4 terms is within $\frac{1}{1000}$,

Hence

$$\arctan \frac{1}{2} \approx \frac{1}{2} - \frac{1}{24} + \frac{1}{160} - \frac{1}{896}$$

$$\approx 0.4634672619$$

Calc Check

$\left|\arctan\left(\frac{1}{2}\right) - P_4\left(\frac{1}{2}\right)\right| = .00018 < .001$

Alt Method

$$|a_{n+1}| = \frac{1}{2^{(2n+1)}(2n+1)} \le \frac{1}{1000}$$

$$2^{2n+1}(2n+1) \ge 1000$$

$n \ge 3.066$

So 1st 4 terms